Statistica Sinica 12(2002), 599-607

TOTAL BEHAVIOR OF LIKELIHOOD DISPLACEMENT

Wai-Yin Poon and Yat Sun Poon

The Chinese University of Hong Kong and University of California at Riverside

Abstract: Likelihood displacement is a useful measure of influence. Ideally, we would like to have a complete influence graph of the likelihood displacement. As complete graphs cannot be obtained easily in most situations, previous works have considered methods for extracting critical information contained in the graphs. Emphases have been placed on characterizing the graphs at specific points, such as the optimal point or the boundary points. In this paper, we adopt a different perspective and develop measures which describe the total behavior of a likelihood displacement function. Two measures, namely the standardized arc-length and the mean displacement, are constructed and their applications are illustrated with the case-weights linear regression model.

Key words and phrases: Likelihood displacement, linear regression, mean displacement, standardized arc-length, total displacement.

1. Introduction

Likelihood displacement is a very important concept as it provides a general approach to study the problem of influence. Let β be a $p \times 1$ vector of unknown parameters and $\hat{\beta}$ be the maximum likelihood (ML) estimate of β obtained from a sample of size n. The influence of the ith observation on the parameter estimate can be assessed by studying the difference between $\hat{\beta}$ and $\hat{\beta}_{(i)}$, where $\hat{\beta}_{(i)}$ denotes the ML estimate of β obtained from the sample of size $n-1$ excluding the ith observation. Likelihood displacement, which is defined as $LD_i = 2(L(\hat{\beta}) - L(\hat{\beta}_{(i)}))$, where L denotes the log-likelihood, is a popular measure for assessing the influence of case i. In fact, in the linear regression model $y = X\beta + \epsilon$, where y is a $n \times 1$ vector, X is a $n \times p$ matrix, $E(\epsilon) = 0$ and $Var(\epsilon) = \sigma^2 I$, Cook's distance $D_i = (\hat{\beta} - \hat{\beta}_{(i)})^T X^T X(\hat{\beta} - \hat{\beta}_{(i)})/p\sigma^2$ is a function of LD_i (Cook (1977), Cook, Pena and Weisberg (1988), Equation (4)).

Another assessment by perturbation is also prominent in influence analysis. In this approach, diagnostics are obtained from local changes of relevant measures caused by small perturbations (see, e.g., Belsley, Kuh and Welsch (1980), Pregibon (1981), Cook (1986), Poon and Poon (1999)). Let $\omega = (\omega_1, \ldots, \omega_n)^T$ in $\Delta \subseteq R^n$ be the vector of perturbation parameters, Δ a set of relevant perturbations, c_0 be a point in the domain of perturbation representing the null

TOTAL BEHAVIOR OF LIKELIHOOD DISPLACEMENT

Wai-Yin Poon and Yat Sun Poon

Reprinted from
Statistica Sinica Vol. 12, No. 2, pp. 599-607
April 2002

Statistica Sinica 12(2002), 599-607

TOTAL BEHAVIOR OF LIKELIHOOD DISPLACEMENT

Wai-Yin Poon and Yat Sun Poon

The Chinese University of Hong Kong and University of California at Riverside

Abstract: Likelihood displacement is a useful measure of influence. Ideally, we would like to have a complete influence graph of the likelihood displacement. As complete graphs cannot be obtained easily in most situations, previous works have considered methods for extracting critical information contained in the graphs. Emphases have been placed on characterizing the graphs at specific points, such as the optimal point or the boundary points. In this paper, we adopt a different perspective and develop measures which describe the total behavior of a likelihood displacement function. Two measures, namely the standardized arc-length and the mean displacement, are constructed and their applications are illustrated with the case-weights linear regression model.

Key words and phrases: Likelihood displacement, linear regression, mean displacement, standardized arc-length, total displacement.

1. Introduction

Likelihood displacement is a very important concept as it provides a general approach to study the problem of influence. Let β be a $p \times 1$ vector of unknown parameters and $\hat{\beta}$ be the maximum likelihood (ML) estimate of β obtained from a sample of size n. The influence of the ith observation on the parameter estimate can be assessed by studying the difference between $\hat{\beta}$ and $\hat{\beta}_{(i)}$, where $\hat{\beta}_{(i)}$ denotes the ML estimate of β obtained from the sample of size $n-1$ excluding the ith observation. Likelihood displacement, which is defined as $LD_i = 2(L(\hat{\beta}) - L(\hat{\beta}_{(i)}))$, where L denotes the log-likelihood, is a popular measure for assessing the influence of case i. In fact, in the linear regression model $y = X\beta + \epsilon$, where y is a $n \times 1$ vector, X is a $n \times p$ matrix, $E(\epsilon) = 0$ and $Var(\epsilon) = \sigma^2 I$, Cook's distance $D_i = (\hat{\beta} - \hat{\beta}_{(i)})^T X^T X(\hat{\beta} - \hat{\beta}_{(i)})/p\sigma^2$ is a function of LD_i (Cook (1977), Cook, Pena and Weisberg (1988), Equation (4)).

Another assessment by perturbation is also prominent in influence analysis. In this approach, diagnostics are obtained from local changes of relevant measures caused by small perturbations (see, e.g., Belsley, Kuh and Welsch (1980), Pregibon (1981), Cook (1986), Poon and Poon (1999)). Let $\omega = (\omega_1, \ldots, \omega_n)^T$ in $\Delta \subseteq R^n$ be the vector of perturbation parameters, Δ a set of relevant perturbations, c_0 be a point in the domain of perturbation representing the null perturbation, and $\hat{\beta}_\omega$ be the ML estimate of the perturbed model with likelihood $L(\beta|\omega)$. Diagnostic measures are developed by studying the behavior of $f(\omega)$, a generalization of the likelihood displacement LD_i, where (Cook, (1986))

$$f(\omega) = LD(\omega) = 2(L(\hat{\beta}|c_0) - L(\hat{\beta}_\omega|c_0)) = 2(L(\hat{\beta}) - L(\hat{\beta}_\omega)), \tag{1}$$

and differentiation (usually at c_0) is used to replace the role of differencing in the deletion approach.

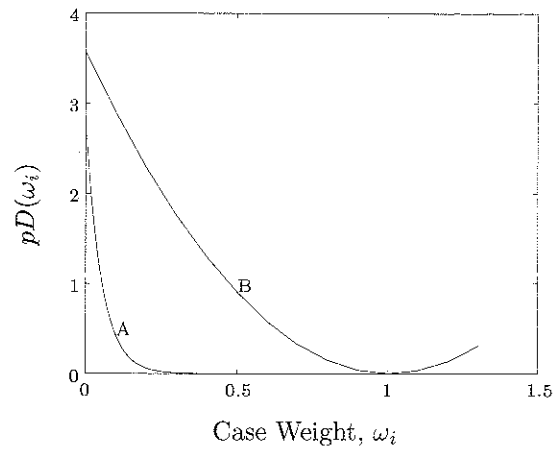

Figure 1. Figure 1 of Cook (1986).

The likelihood displacement is a unifying concept in influence analysis (see also the discussion in Cook, Pena and Weisberg (1988)). Different information is revealed by studying f at various points ω. Cook illustrated this issue by using Figure 1 (Cook (1986), his Figure 1 with modification in notation) under the case-weights linear regression model. Let $\hat{\beta}_\omega$, $\omega = (\omega_1, \ldots, \omega_n)^T$, be the estimate of β in the linear regression model when the ith case has weight ω_i, and $D(\omega) = (\hat{\beta} - \hat{\beta}_\omega)^T X^T X(\hat{\beta} - \hat{\beta}_\omega)/p\sigma^2$ be the general version of the Cook's distance D_i. It can be shown that $pD(\omega) = f(\omega)$ (see (4) of Cook (1986)). When the ith case has weight ω_i and the remaining cases have weight 1, one has a function of ω_i, $pD(\omega_i)$, say. Note that when $\omega_i = 0, D(\omega_i)$ is the Cook's distance of Case i. Suppose for two observations, $i = A, B$, the graphs for $pD(\omega_i)$ are given as in Figure 1. More details about the graphs are available in Cook (1986) and, in particular, if Figure 1 is magnified substantially, it will be found that $pD(0) = 3.5$ for both Case A and Case B (Cook (1986, p.142)). As a result, they are judged to be equally influential when using Cook's distance. However since the value of $pD(\omega_i)$ for Case B is everywhere greater than that of A, and the difference is substantial for some ω_i, it is necessary to investigate the behaviour of $pD(\omega_i)$ at values other than at the boundary. In view of this, measures of influence characterizing the behaviour of $f(\omega)$ over the entire range of ω are preferred in order to obtain a complete understanding of the influence of a chosen perturbation scheme. Such measures are not easy to construct. The normal curvature (Cook (1986)) assessing the behaviour of $f(\omega)$ at c_0 is one measure that can provide information for $f(\omega)$ different from that provided by Cook's distance.

The objective of this paper is to develop measures to characterize the behavior of the influence graph of the likelihood displacement over the entire perturbation range. In Section 2 we introduce two measures, the standardized arc-length and the mean displacement. Applications of these measures are discussed and illustrated with the case-weights linear regression model. Examples based on data sets in the literature are used for analysis and the results are reported. In Section 3, we consider the simultaneous influences of two or more perturbation parameters and further discussion is in Section 4.

2. Total Measures of Likelihood Displacement

2.1. Standardized arc-length

Assume the perturbation parameters are defined in a box: $B = \{\omega = (\omega_1, \ldots, \omega_n)^T : u_i \le \omega_i \le v_i, \text{ for all } 1 \le i \le n\}$. The graph of the function f in (1) is called the influence graph. When ℓ is a vector in R^n, we consider a perturbation in the direction ℓ and the influence graph over the line $\omega_\ell(t) = c_0 + t\ell$. For simplicity, denote $f(\omega_\ell(t))$ by $f_\ell(t)$. When t varies from 0 to a point d, denote $\omega_\ell(d)$ by c_1. The arc-length of the graph of f from $f_\ell(0)$ to $f_\ell(d)$ over the path $\omega_\ell(t)$ is $\int_0^d (|\ell|^2 + (df_\ell(t)/dt)^2)^{1/2} dt$, where $|\ell|$ is the norm of the vector ℓ. Arc-length is a measure reflecting aggregate local effects such as curvature. For example, a curve without curvature anywhere is a straight line. Since the displacement function satisfies $f_\ell(0) = 0$ and has a minimum at $t = 0$, the normal curvature of the graph of f along the line $\omega_\ell(t)$ is identically zero only if $f_\ell(t) = 0$ for all t. In this case, the arc-length is $d|\ell|$. Hence a natural benchmark which reflects the deviation of the graph of $f_\ell(t)$ from a horizontal straight line is the displacement in parameter, i.e., $d|\ell| = |c_1 - c_0|$.

Definition 1. The standardized arc-length of the graph of the function along the line $\omega_\ell(t) = c_0 + t\ell$ from c_0 to $c_1 = c_0 + d\ell$ is

$$P(c_0, c_1) = \frac{\int_0^d \sqrt{|\ell|^2 + \left(\frac{df_\ell(t)}{dt}\right)^2}\, dt}{|c_1 - c_0|}. \tag{2}$$

Thus $P(c_0, c_1) \geq 1$ and $P(c_0, c_1)$ close to 1 indicates the model is insensitive to the perturbation within the specific range in the chosen direction.

When $\{e_1, \ldots, e_n\}$ is the standard basis of the perturbation domain, we denote the standardized arc-length from c_0 to $c_0 - e_i$ by P_i. It measures the aggregate curvatures of the ith perturbation parameter ω_i for t varying from 0 to 1 along $-e_i$, and provides information about the stability of an analysis with respect to the perturbation of the ith perturbation parameter. For example, the influence graphs in Figure 1 are obtained by perturbing the weights of Case A and Case B in a case-weights linear regression model, and are the influence graphs along the directions $-e_A$ and $-e_B$. If $0 \leq \omega_i \leq 1$, then $c_0 = (1, \ldots, 1)^T$ and $c_1 = (1, \ldots, 0, \ldots, 1)^T$, where 0 is at the Ath or Bth slot, and t varies from 0 to 1. Although the values of Cook's distance for Case A and Case B are the same, reflecting identical deletion influence of these two cases, we obtain $P_A > P_B$ as shown in Section 2.4. The larger P_A suggests that there are points in the graph of A with larger curvature than those of B. In other words, if we perturb the weights instead of deleting the cases, the influences of Cases A and B can be very different.

2.2. Total displacement

On the other hand, based on the observation from Figure 1 that the value of $f(\omega)$ for Case B is bigger than that of Case A over the entire range of perturbation, one may conclude that perturbations in the weight attached to Case B are more sensitive than those of A (see the discussion in Cook (1986, p.135)). This conclusion is based on the concept that sensitivities are measured by deviations with respect to the null (un-perturbed) model. We can quantify this concept, *total displacement*, by the integral $T(c_0, c_1) = \int_0^d f_\ell(t)dt$. If perturbation does not induce noticeable change in the ML estimates of the parameters over its entire range, the value of the total displacement is small, say close to zero.

Definition 2. The mean displacement along the line $\omega_\ell(t) = c_0 + t\ell$ from c_0 to $c_1 = c_0 + d\ell$ is

$$M(c_0, c_1) = \frac{1}{|c_1 - c_0|} \int_0^d f_\ell(t)dt. \tag{3}$$

Clearly, if $|c_1 - c_0| = 1$, the mean displacement is equal to the total displacement.

We denote the total displacement from c_0 to $c_0 - e_i$ by T_i and the mean displacement by M_i. For Cases A and B in Figure 1, we obtain $M_A = T_A$=0.163 and $M_B = T_B$=1.167 as shown in Section 2.4. That is, the total displacement for Case A is smaller than that of Case B.

2.3. Case-weights perturbation in linear regression

In this section, we illustrate the proposed measures with the case-weights linear regression model. A discussion of this model is available in Cook (1986) and Lawrance (1991). Let Ω be the diagonal matrix whose ith entry ω_i is a non-negative number. When σ^2 is known, the log-likelihood function for the model with case-weights perturbation is given by $L(\beta|\omega) = -\frac{1}{2\sigma^2}\sum_{i=1}^n \omega_i(y_i - x_i^T\beta)^2 = -\frac{1}{2\sigma^2}(y - X\beta)^T\Omega(y - X\beta)$, where x_i^T is the ith row of the matrix X. The un-perturbed model is obtained by setting $\omega{=}c_0 = (1,\ldots,1)^T$. When $H_\omega = X(X^T\Omega X)^{-1}X^T\Omega$, then $H_{c_0} = X(X^TX)^{-1}X^T$ is the usual hat matrix. Moreover, $\hat{\beta}_\omega = (X^T\Omega X)^{-1}X^T\Omega y$ and $X\hat{\beta}_\omega = H_\omega y$. The displacement of the log-likelihood is

$$f(\omega) = \frac{1}{\sigma^2}\left(|y - H_\omega y|^2 - |y - Hy|^2\right) = \frac{1}{\sigma^2}|Hy - H_\omega y|^2. \qquad (4)$$

We perturb the model from c_0 along the direction $\ell = -e_i$ to $c_0 - e_i$. Denote the restriction of the likelihood displacement along this line by $f_i(t), 0 \le t \le 1$. To calculate this function, let H_i be the ith column of the matrix H, h_i be the ith diagonal element of H, and $\Omega = I - te_ie_i^T$. Using Atkinson (1985, equation (2.2.1)) we have

$$H_\omega = H - \frac{t}{1 - th_i}H_i(e_i^T - H_i^T). \qquad (5)$$

By (4), $f_i(t) = \frac{t^2r_i^2h_i}{\sigma^2(1-th_i)^2}$ and $\frac{df_i(t)}{dt} = \frac{2}{\sigma^2}\frac{tr_i^2h_i}{(1-th_i)^3}$, where r_i denotes the least square residual of the ith case. Therefore, the standardized arc-length and the total displacement for perturbing the weight of the ith case are

$$P_i = \int_0^1 \sqrt{1 + \frac{4}{\sigma^4}\frac{t^2r_i^4h_i^2}{(1 - th_i)^6}}dt \quad \text{and} \quad M_i = T_i = \frac{r_i^2h_i}{\sigma^2}\int_0^1 \frac{t^2}{(1 - th_i)^2}dt, \quad (6)$$

respectively. One can employ, for example, the IMSL (1994, p.686) subroutine DQDAGS to compute P_i and T_i for given r_i, h_i and σ^2.

Formula (6) implies that $P_i \ge 1$; and $P_i = 1$ if and only if the leverage h_i or the residual r_i is equal to zero. Similarly, $T_i \ge 0$, and $T_i = 0$ if and only if the leverage or the residual is equal to zero. After perturbation, the leverage of the ith case is given by the ith diagonal term of H_ω, and the vector of residuals of the perturbed model $r(t)$ is equal to $(I - H_\omega)y$. By (5), they are respectively given by $h_i(t) = \left(\frac{1-t}{1-th_i}\right)h_i$ and $r_i(t) = \frac{r_i}{1-th_i}$. Therefore, $h_i(t)$ is identically zero when $h_i = 0$, and the ith observation of y has no contribution to the predicted value of y throughout the perturbation space. Moreover, when $r_i = 0$, the ith case lies on the regression line throughout the perturbation space.

2.4. Examples

Cook's Example. For the example in Figure 1, information about the leverage h_i and the normal curvature $C_i = 2r_i^2 h_i/\sigma^2$ for Cases A and B is available in Cook (1986, p.142). These are $h_A = 0.95$, h_B=0.01, C_A=0.02 and $C_B = 6.9$. Substituting these values into (6), we find P_A=4.739, P_B=3.749, T_A=0.163 and T_B=1.167. The interpretation of these values has been given in Section 2.1 and Section 2.2.

Paul and Fung's Data. The second data set is taken from Paul and Fung (1991, Example 2). Figure 2 presents the scatter plot of the data. Cases 7, 8 and 9 are atypical. The values for P_i, T_i and Cook's distance D_i are presented in Table 1 according to size. All measures identify Cases 9 and 7 as observations that worth further attention: P_7 and P_9 are significantly different from 1; T_7 and T_9 are significantly different from 0; D_7 and D_9 are relatively large.

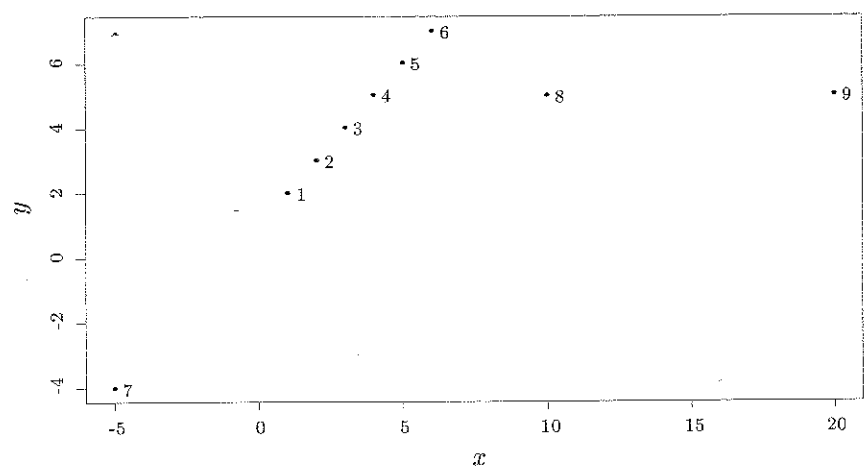

Figure 2. Scatter plot of Paul and Fung's data set. (Paul and Fung (1991))

Table 1. Influential measures for Paul and Fung's data set.

Case i	9	7	6	5	4	3	1	2	8
P_i	10.100	3.309	1.024	1.008	1.002	1.000	1.000	1.000	1.000
T_i	1.636	0.771	0.059	0.034	0.018	0.007	0.002	0.001	0.000
D_i	4.910	1.493	0.093	0.055	0.028	0.011	0.003	0.001	0.000

3. Total Measures of Displacement with Multiple Parameters

3.1. Total measures

The measures P_i and T_i developed in the last section are concerned with influence of individual perturbation parameters. It is natural to consider the joint influence over a set of $q \leq n$ parameters: $\omega(t_1, \ldots, t_q) = c_0 - t_1 e_1 - \ldots - t_q e_q$, with $0 \leq t_i \leq d_i$. The generalization of the measures defined in (2) and (3) are as follows.

Definition 3. The standardized volume of the influence graph is

$$\frac{1}{d_1 \cdots d_q} \int_0^{d_1} \cdots \int_0^{d_q} \sqrt{1 + \left(\frac{\partial f}{\partial t_1}\right)^2 + \cdots + \left(\frac{\partial f}{\partial t_q}\right)^2} dt_1 \cdots dt_q. \tag{7}$$

Definition 4. The total displacement and the mean displacement are given, respectively, by

$$\int_0^{d_1} \ldots \int_0^{d_q} f(\omega) dt_1 \ldots dt_q, \quad \text{and} \quad \frac{1}{d_1 \ldots d_q} \int_0^{d_1} \ldots \int_0^{d_q} f(\omega) dt_1 \ldots dt_q. \tag{8}$$

We denote the standardized volume and the total displacement of the perturbation

$$\omega(t_i, t_j) = c_0 - t_i e_i - t_j e_j, \qquad 0 \leq t_i \leq d_i, \qquad 0 \leq t_j \leq d_j, \tag{9}$$

by P_{ij} and T_{ij} respectively.

3.2. Joint influence in linear regression

In this section, we continue using case-weights linear regression to illustrate our theory. Set $\Lambda = 1 - (t_i h_i + t_j h_j) + t_i t_j (h_i h_j - h_{ij}^2)$, where h_{ij} is the (i,j)th entry in H, then as in Section 2.3 based on the perturbation in (9),

$$f = \frac{1}{\sigma^2 \Lambda^2} \sum_{1 \leq i,j \leq n} \Big\{ t_i^2 h_i \left(r_i(1 - t_j h_j) + r_j t_j h_{ij}\right)^2 + t_j^2 h_j \left(r_j(1 - t_i h_i) + r_i t_i h_{ij}\right)^2$$
$$+ 2 t_i t_j h_{ij} \left(r_i(1 - t_j h_j) + r_j t_j h_{ij}\right) \left(r_j(1 - t_i h_i) + r_i t_i h_{ij}\right) \Big\}.$$

It also follows that

$$\frac{\partial f}{\partial t_i} = \frac{2}{\sigma^2 \Lambda^3} \sum_{1 \leq j \leq n} \Big\{ t_i \left(r_i(1 - t_j h_j) + r_j t_j h_{ij}\right)^2 \left(h_i - t_j (h_i h_j - h_{ij}^2)\right)$$
$$+ t_j h_{ij} \left(r_i(1 - t_j h_j) + r_j t_j h_{ij}\right) \left(r_j(1 - t_i h_i) + r_i t_i h_{ij}\right) \Big\}. \tag{10}$$

Here one can interchange the indices i and j to obtain $\frac{\partial f}{\partial t_j}$ and then compute the standardized area and the total displacement through the definitions. The IMSL (1994, p.721) subroutine DQAND can be employed to do the computations.

Paul and Fung's Data. The standardized area P_{ij} of perturbing the weights of two cases have been computed, see Table 2. It is found that any P_{ij} where i or j is 7 or 9 deviates substantially from 1, and P_{89} is found to be 18.5, indicating a strong joint influence of these two cases.

Table 2. Standardized area for Paul and Fung's data.

Case	1	2	3	4	5	6	7	8
2	1.000							
3	1.000	1.001						
4	1.002	1.005	1.011					
5	1.008	1.014	1.027	1.049				
6	1.024	1.036	1.057	1.091	1.143			
7	4.339	3.781	3.420	3.192	3.058	2.998		
8	1.000	1.000	1.000	1.003	1.010	1.032	3.310	
9	10.12	10.12	10.00	9.778	9.450	9.019	8.112	18.50

4. Discussiôn

Standardized arc-length and the total displacement are defined along general directions. Therefore, one may investigate the total behavior of the likelihood displacement for any particular direction, for example the maximum eigenvalue direction emphasized by Cook (1986). Our attention is focused on the basis $\{e_1, \ldots, e_n\}$ because of Poon and Poon (1999, Theorem 4). They developed a relation between the normal curvature of e_i and all influential eigenvectors at c_0 under the assumption that the Hessian matrix of the influence graph is semi-positive definite, and concluded that the normal curvature along e_i is an effective measure for assessing local influence.

Our development has not taken into account the effect of nuisance parameters. When an unknown parameter is of no interest, we assume in our development that it is known, and in computation we replace it by the usual maximum likelihood estimate. Cook (1986, equation(7)) defined an influence graph $LD_s(\omega)$ to replace the usual influence graph in the presence of nuisance parameters. The development in this paper can be generalized with $f(\omega)$ replaced by $LD_s(\omega)$ to address precisely the effect of nuisance parameters.

Acknowledgements

The work in this paper was partially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (RGC Ref. No. CUHK4186/98P). Y.S. Poon thanks the Department of Statistics at the Chinese University of Hong Kong for hospitality. Thanks are due the referees, the

Associate Editor and the Editor for their useful comments, and Michael Leung for his assistance in the preparation of the Figures.

References

Atkinson, A. C. (1985). *Plots, Transformations and Regression*. Clarendon Press, Oxford.

Belsley, D. A., Kuh, E. and Welsch, R. E. (1980). *Regression Diagnostics*. Wiley, New York.

Cook, R. D. (1977). Detection of influential observation in linear regression. *Technometrics* **19**, 15-18.

Cook, R. D. (1986). Assessment of local influence (with discussion). *J. Roy. Statist. Soc. Ser. B* **48**, 133-169.

Cook, R. D., Pena, D. and Weisberg, S. (1988). The likelihood displacement: An unifying principle for influence measures. *Comm. Statist. Theory Methods* **17**, 623-640.

IMSL Math/Library (1994). *FORTRAN Subroutines for Mathematical Applications*. Visual Numerics, Inc., Houston.

Lawrance, A. J. (1991). Local and deletion influence. In *Directions in Robust Statistics and Diagnostics* (Edited by W. Stahel and S. Weisberg), Part 1, pp.141-157. Springer, Berlin.

Paul, S. R. and Fung, K. Y. (1991). A generalized extreme studentized residual multiple-outlier-detection procedure in linear regression. *Technometrics* **33**, 339-348.

Poon, W. Y. and Poon, Y. S. (1999). Conformal normal curvature and assessment of local influence. *J. Roy. Statist. Soc. Ser. B* **61**, 51-61.

Pregibon, D. (1981). Logistic regression diagnostics. *Ann. Statist.* **9**, 705-724.

Department of Statistics, The Chinese University of Hong Kong, Shatin, Hong Kong.
E-mail: wypoon@cuhk.edu.hk

Department of Mathematics, University of California at Riverside, Riverside, CA92521, U.S.A.
E-mail: ypoon@math.ucr.edu

(Received March 2000; accepted September 2001)