



Special section: 60 years of Calabi conjecture

In search for canonical metrics on Kähler manifolds, in the mid-1950s, Eugenio Calabi proposed his celebrated conjecture on the existence of a unique Kähler metric of which the Ricci form is a prescribed $(1, 1)$ -form representing the first Chern class of a Kähler manifold. It is especially important when the first Chern class vanishes, where Calabi conjecture implies the existence of a unique Ricci-flat metric in every Kähler class. For this case Calabi conjecture was proven by Shing-Tung Yau in 1976, and Kähler manifolds with vanishing first Chern classes are conventionally referred to as Calabi–Yau manifolds.

Yau’s solution of Calabi conjecture triggered the study of Calabi–Yau manifolds in both mathematics and physics. Classically, they give rise to various nontrivial vacuum solutions of the Einstein equation. In addition, its role in physics was explored along with the evolution of superstring theory. In 1985, Candelas, Horowitz, Strominger and Witten discovered that Calabi–Yau manifolds have the form of the extra six “unseen” dimensions in superstring theory to admit $N = 1$ supersymmetry in effective four dimensional space–time. This inspired a great amount of works on the search for examples of Calabi–Yau manifolds or their generalizations arising from flux compactification in string theory.

In the late 1980s, physicists including Dixon, Lerche, Vafa, and Warner observed that the same $N = 2$ superconformal field theory can be realized by sigma models on two different Calabi–Yau manifolds. This led to the suggestion that Calabi–Yau manifolds should come in pairs. A class of such examples for superstring models were soon found by Greene and Plesser, who named them mirror pairs. In 1992, Candelas, de la Ossa, Green and Parkes made a breakthrough study on the example of the mirror quintic pair. They showed that the long-standing enumerative problem of counting rational curves on generic quintic 3-folds can be solved by studying the variation of Hodge structures on the mirror. This surprising result greatly increased the interest in mirror symmetry between Calabi–Yau manifolds in both the mathematics and the physics community. The content of mirror symmetry was further expanded in 1994 when Kontsevich proposed homological mirror symmetry by bringing in categorical structures and D-branes. In 1997, based on a deep insight via D-brane considerations, Strominger, Yau and Zaslow proposed a geomet-

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ric approach to mirror manifolds via dual special Lagrangian tori fibrations. Nowadays, mirror symmetry has become an extremely fruitful idea as a duality between symplectic geometry and complex geometry as well as a testing ground for string models. It leads to rather deep research in geometry, topology and even arithmetic.

The current special section celebrating 60 years of Calabi conjecture collects frontier research on Calabi–Yau manifolds related to physics, in particular to string theory. Various topics, as mentioned above, are covered. It aims to present the modern scope of the beauty of Calabi–Yau geometry and its deep role in physics applications.

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